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Main Features of Designing With Brittle Materials

D. Rubeša, B. Smoljan, and R. Danzer

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Brittle material behavior and mode of failure are contrasted with those characteristics of ductile materials. The stochastic nature of brittle fracture, which results from the random occurrence of fracture-initiating microstructural imperfections, necessitates a probabilistic fracture mechanics approach to design with brittle materials. It is also clearly shown which main properties of brittle materials have to be optimized to improve the reliability of mechanically loaded components made of brittle materials. Important features of designing with brittle materials are elucidated and illustrated by an exemplary design calculation of a ceramic disc spring. It is shown how even environmentally induced subcritical crack growth, characteristic of ceramic materials, can be adequately accounted for in the assessment of reliability.

Keywords brittle material, ceramics, designing with brittle material, probabilistic fracture mechanics

1. Introduction

Throughout history, brittle materials have been used for making mechanically loaded components and devices. In fact, the earliest materials for ancient tools, e.g., natural stone and bone, were brittle. One of the major materials for engineering construction at the beginning of the industrial era was brittle cast iron, of which the first metal bridges were built, also. However, much skill and experience, rather than a methodology, were involved in designing with brittle materials.

The advances in the production of steel and the development of other ductile alloys in the last one hundred years or so elevated them to the main structural materials. This coincided with the foundation and development of design methodologies, and particularly the related calculation of strength. Hence, today's well-established design practices are generally appropriate only to ductile materials.[1]

The in-service demands made on structural components and machine parts often require the application of brittle materials, particularly ceramics and glasses, due to certain other outstanding properties such as high-temperature stability, oxidation and corrosion resistance, dimensional stability, hardness and wear resistance, or other special thermal, electrical, or optical properties. A special approach to design and the ever-improving mechanical properties of modern engineering ceramics enable their safe and optimal use as structural materials. Many examples of the successful and beneficial application of engineer-

D. Rubeša, University of Applied Sciences (FH Joanneum), Dept. of Automotive Engineering, Alte Poststr. 149, A-8020 Graz, Austria; **B. Smoljan,** University of Rijeka, Faculty of Engineering, Vukovarska 58, HR51000 Rijeka, Croatia; and **R. Danzer,** University of Leoben, Dept. of Structural and Functional Ceramics, Peter-Tunner-Str., A-8700 Leoben, Austria. Contact e-mail: bozo.smoljan@ri.hinet.hr.

[1]This does not apply to civil engineering. The major building materials of today, namely cement, concrete, and clay also are brittle, but engineers have learned how to use them primarily in compression, in which they are not as prone to failure.

Nomenclature

A	parameter in the approximation of the distribution of flaw lengths
C	parameter of subcritical crack growth
C_1	parameter of subcritical crack growth
E	modulus of elasticity
E'	E for plane strain, $E/(1 - \nu)$ for plane stress
F	applied force
K_I	stress intensity factor for mode I loading
K_{Ic}	fracture toughness
N_c	mean number of critical flaws within a component
P_f	probability of failure
P_s	probability of survival
V	volume
V_0	reference (or unit) volume
Y	geometry constant
a	crack length
a_c	critical crack length
a_i	initial crack length
g	frequency distribution density function of flaw lengths
m	Weibull modulus
n	parameter of subcritical crack growth
p	parameter in the approximation of the distribution of flaw lengths
$\mathbf{r}$	position vector
t	time
Γ	gamma-function
Γ_c	energy expended per unit area of crack advancement
ν	Poisson's ratio
σ	applied stress
σ_0	characteristic strength
σ_{3b}	flexural (or bending) strength determined by three-point bending
σ_{4b}	flexural (or bending) strength determined by four-point bending
σ_f	fracture stress, i.e. tensile strength
σ_χ	meridian stress in a disc spring
σ_φ	circumferential stress in a disc spring
$\bar{\sigma}$	equivalent stress
τ_{xz}	shear stress in a disc spring

ing ceramics for mechanically loaded components, especially at high temperatures and in a hostile environment, are already known: from bearings, dies and cutting tools, to valves for internal combustion engines, gas turbine components, and also, bio-implants.

The necessity for an essentially different design practice for brittle materials based on the probabilistic calculation of strength, as compared with that generally applied to conventional, ductile materials, comes from the different modes and criteria of failure in these two types of materials. This paper reveals the reason and presents the methodology for the probabilistic assessment of the reliability of load-bearing components made of brittle materials. The consequent peculiarities of designing with brittle materials are clearly pointed out. These are finally illustrated using the simple and clear example of a ceramic disc spring.

2. Fracture and Strength of Ductile and Brittle Materials

2.1 Distinction Between Ductile and Brittle Material Behavior and Fracture

The response up to the point of failure of a material under an applied load can be either purely elastic, or it involves a permanent deformation. This is closely connected to the mode of fracture, which in the first case is brittle, and in the latter case is ductile. Accordingly, materials are also classified as being either brittle or ductile.

The factor that decides which of the two main types of material behavior and fracture takes place in crystalline solids at low and moderate temperatures is the mobility of dislocations. In metallic materials the nature of atomic bonding and relatively simple crystal structures lead to the generally high mobility of dislocations driven by stresses above a certain limit, i.e., the yield strength. This results in general plasticity and strengthening (work-hardening) prior to fracture, and hence, metals are typically ductile, although most of them also become brittle below a certain transition temperature. Ductile fracture proceeds after voids, which either pre-exist or have nucleated within the material, have grown by plastic flow so much that they link and give a fracture path. In the case of macroscopically localized plastic deformation, ductile fracture may also result from the loss of stability of a loaded component. Although the micromechanisms of ductile fracture are rather complex and influenced by numerous factors, and the phenomenological elasto-plastic deformation behavior of ductile material is essentially more difficult to model than elasticity, the relative determinacy of the macroscopic stress-strain response and of the values of characteristic strength (e.g., the yield strength and ultimate tensile strength) enable a deterministic approach to the calculation of strength and to design.

In contrast to metallic materials, the motion of dislocations in ionically and covalently bonded materials with complex crystal structures is almost thoroughly inhibited, which leads to extremely high values of yield strength. This is why ceramic materials, for instance, are inherently brittle. Generally, materials are, or become, brittle whenever stress relaxation by the internal redistribution of material, e.g., by dislocation motion, is sufficiently restrained. In most metals this happens at low enough temperatures but can also be caused by the presence of alloying elements, environmentally assisted hydrogen attack, or neutron irradiation. Thus, before the yield strength can be reached, brittle materials fail either by transgranular cleavage or by brittle intergranular fracture (BIF) in a catastrophic way without any or with negligible macroscopic permanent deformation. Cleavage or BIF is induced by a crack that under a certain tensile stress becomes unstable and propagates at the speed of sound between crystal planes or grains by successively breaking interatomic bonds. Minute internal and surface crack-like flaws are an inevitable consequence of the manufacturing process but can also result from a thermal shock or inappropriate handling; other microstructural defects can also turn into microcracks under the action of stress. Cracks of any size cause stress concentrations at their tips. Since a redistribution (relaxation) of stress by plastic flow is not possible, due for example to the lack of dislocation mobility, the stress can locally reach the cohesive strength and tear the atomic bonds causing the crack to propagate. The form, density, and distribution, and most critically, the size of the inherent microscopic flaws are the decisive factors for the onset of fracture under an applied load and thus for the strength of a brittle material. Since the occurrence of microstructural defects is of a stochastic nature, so is brittle fracture; and hence the strength of a brittle material is an intrinsically statistical quantity. This is confirmed by the large scatter of experimental data. Consequently, design with brittle materials requires a probabilistic approach.

2.2 Criterion for the Actual Fracture Mode

Two different failure modes compete in any material/component with increasing applied stress level; either fast fracture will happen by sudden crack propagation, or the yield strength will be reached first and further increase in the applied stress will cause the material to fail in a ductile manner.

According to the modified Griffith energy-balance criterion, assuming linear elastic material behavior, a crack will catastrophically propagate under a uniform applied tensile loading normal to the crack plane if a certain combination of the applied stress σ and the crack length a reaches a critical value, i.e., if

$$\sigma a^{1/2} = \frac{1}{Y}(E'\Gamma_c)^{1/2} \qquad \text{(Eq 1)}$$

where E' is the modulus of elasticity, E, for plane stress and $E/(1 - \nu^2)$ for plane strain conditions, with ν as Poisson's ratio, Γ_c as the energy absorbed (or expended) per unit area of crack advancement, and Y as a dimensionless constant of the order of unity that depends on the crack configuration and geometry. Thus the resistance of a material to crack propagation is determined by the material property

$$K_{Ic} = (E'\Gamma_c)^{1/2} \qquad \text{(Eq 2)}$$

called the fracture toughness. It can also be regarded as the critical value of the stress intensity factor $K_I = Y\sigma a^{1/2}$ that characterizes the stress field near the tip of a crack in a linearly elastic solid under mode I (normal to the crack plane) loading.

Since the extremely high values of yield strength in brittle materials are difficult even locally to reach, virtually no dissipation of energy due to plastic flow is possible, and the energy required for crack propagation is essentially needed only for the formation of new fracture surfaces. Hence these materials have intrinsically low fracture toughness. Inherent microstructural defects are then always large enough to allow the failure condition (Eq 1) for fast fracture to become critical and determinative for strength. It should be noted that this failure criterion does not apply for compressive stresses. These are not directly harmful for inherent flaws, and brittle materials in compression fail by a different mechanism.[1] This, together with the high resistance to yielding, results in compressive strengths of brittle materials being one order of magnitude higher than tensile strengths and generally even better than those of ductile materials. Appropriate design should take advantage of this property.

The fracture toughness of ductile materials, and thus the resistance to sudden crack propagation, is substantially higher than that of brittle materials because the yield strength is reached in a wide region around the crack tip, and a lot of energy is absorbed by plastic flow during crack propagation. In fact, a crack in this case does not propagate by cleavage but by the coalescence of microvoids nucleated in front of the crack. A sharp crack also tends to blunt by plastic flow so that crack propagation may even be arrested if the applied stress is not increased. Excessive plasticity requires corrections to be introduced to linear elastic fracture mechanics theory and may indeed render it inapplicable. Anyway, the characteristically low yield strength of ductile materials enables general yielding to occur at an applied stress far below that needed for the stress intensity factor to reach the critical value for the typical size of inherent flaws. Therefore, a ductile material can fail by fast crack propagation only if a crack much longer (of a different order of magnitude) than inherent flaws exists. This can be a notch, a flaw in welding, or a fatigue crack.

2.3 Time-Dependent Strength Degradation

Several time-dependent damaging processes can lead to a gradual degradation of strength and thereby to eventual fracture; these include fatigue, creep, oxidation, corrosion, and subcritical crack growth. Their importance differs for the failure of a ductile or a brittle material.

Fatigue under cyclic loading is prominent in metals where localized plastic flow causes cracks to initiate and then to advance step by step with load reversals. The deficiency of plastic flow in brittle materials led to the belief that they do not suffer from fatigue, at least not at low temperatures, although fatigue effects have recently been observed in ceramics as well, but are attributed to other irreversible events.[2,3]

At elevated temperatures failure can happen as a consequence of creep. Again due to the, if not completely inhibited, still rather constrained motion of dislocations, creep in ceramic materials is substantially less pronounced than in metals, and due to higher melting points, it starts at considerably higher temperatures, out of the range of normal technical relevance.[2,4]

The chemical inertness of ceramics, which results in oxidation and corrosion resistance, is one of their most beneficial properties. Metals, on the other hand, are highly susceptible to environmental attack.

Even in the absence of creep and corrosion, brittle fracture of ceramics and glasses still often takes place after they have for some time been subjected to an unaltered loading. This delayed fracture is caused by subcritical crack growth (SCCG), namely the steady growth of cracks smaller than those, which would have led to immediate fracture. These cracks slowly grow under the influence of stress and environment until one reaches the critical size for the applied load, and fracture ensues. The phenomenon is sometimes referred to as static fatigue. This effect is similar to the stress corrosion cracking occasionally observed in metals in a hostile environment. SCCG in ceramics is often ascribed to an analogous mechanism of thermomechanically activated chemical attack of bonds at the crack tip.[5] There are, however, some other possible explanations.[6,7]

3. Probability of Brittle Failure

3.1 Probability of Immediate Brittle Fracture

In the following, the origin of the stochastic nature of strength of brittle materials, the description of which requires a probabilistic approach, will be examined more closely.

According to the criterion for brittle fracture given by Eq 1 and the definition of fracture toughness, Eq 2, the tensile strength σ_f of a brittle material is determined by the relationship

$$\sigma_f = \frac{1}{Y}\frac{K_{Ic}}{a^{1/2}} \qquad \text{(Eq 3)}$$

It depends on the length a of the inherent flaw that will cause fracture. Since the length of randomly distributed flaws within the material is a stochastic variable, the tensile strength is as well: it depends on the probability of finding within the component a crack long enough to cause fracture at a certain stress level.

The failure criterion given by Eq 3 can be rearranged to define the critical flaw size a_c for a given applied tensile stress σ normal to the plane of the flaw:

$$a_c = \left(\frac{1}{Y}\frac{K_{Ic}}{\sigma}\right)^2 \qquad \text{(Eq 4)}$$

Any flaw longer than or equal to a_c will be destructive under the stress σ.

Assuming that a component fails if any one flaw initiates fracture (the weakest link hypothesis), and that there is no interaction between flaws, the probability of failure is equal to the probability of finding at least one critical flaw in the component and is given by

$$P_f = 1 - \exp(-N_c) \qquad \text{(Eq 5)}$$

where N_c is the average number of critical flaws in a large set of identical components.[8] For a homogeneously loaded component and a uniform distribution of flaws of different lengths through the volume, this number equals the local density of critical flaws multiplied by the volume. This implies a size effect: the larger the component, the more likely the critical defect is to be found, and the higher probability of failure. For a nonuniform stress field and/or nonuniformly distributed flaws, the mean number of critical flaws is obtained by integrating the local density of critical flaws over the volume:

$$N_c = \int_V \left(\int_{a_c}^{\infty} g(a,\mathbf{r})\, da \right) dV \quad \text{(Eq 6)}$$

where $g(a,\mathbf{r})$ is the frequency distribution density of flaw lengths at a point defined by the position vector $\mathbf{r}$ [$g(a,\mathbf{r})\,da$ gives the average number of flaws of a length between a and $a + da$ in a unit volume]. Even for a uniform distribution of flaws through the volume, i.e., for $g(a,\mathbf{r}) \equiv g(a)$, in the case of nonhomogeneous loading, the local density of critical flaws given by the inner integral in Eq 6 still varies over the volume since the lower limit of integration depends on the stress at a certain point according to Eq 4.

For a uniform volume distribution of flaws of different lengths, the distribution of flaw lengths in the relevant region of longest cracks (which in fact are decisive for fracture) can be adequately approximated by a simple function of the form

$$g(a,\mathbf{r}) \equiv g(a) = Aa^{-p} \quad \text{(Eq 7)}$$

with adjustable parameters A and p. The inner integral in Eq 6 can then readily be evaluated, and after substituting the expression given by Eq 4, one obtains for the probability of failure the well-known two-parameter Weibull distribution:

$$P_f = 1 - \exp\left(-\frac{1}{V_0} \int_V \left(\frac{\langle\sigma\rangle}{\sigma_0} \right)^m dV \right) \quad \text{(Eq 8)}$$

where m and σ_0 are functionally related to A and p, and V_0 is a certain reference (or unit) volume; the notation

$$\langle\sigma\rangle = \begin{cases} \sigma \text{ for } \sigma \geq 0 \\ 0 \text{ for } \sigma < 0 \end{cases}$$

is used to explicitly indicate that the volume integration should be performed only over the region loaded in tension. Starting from only the weakest link hypothesis, Weibull came to the above distribution heuristically.[9,10] Freudenthal related it to the distribution of fracture initiating flaws within the material, showing that it follows directly from the weakest link hypothesis and a Griffith-type criterion for fracture, for any distribution of flaw lengths which for $a \to \infty$ converges toward zero as fast as a^{-k}, where k is any constant.[11]

The Weibull parameters m and σ_0 can be directly determined as material constants by a statistical evaluation of measured values of strength.[2,12] In this way the distribution of lengths of the most severe flaws (including their random orientation) is indirectly measured as well. A high value of the Weibull modulus m corresponds to small variations in strength, and thus in flaw lengths. Engineering ceramics distinguish themselves by Weibull moduli higher than 10. The other parameter σ_0 represents the characteristic strength; it can be interpreted as the stress at which $(1 - e^{-1}) \cdot 100\% = 63.2\%$ of uniaxially and uniformly loaded samples of the reference volume V_0 will fail.

The above considerations refer to a uniaxial state of stress. For multiaxial stress conditions, an appropriate equivalent (or effective) stress $\overline{\sigma}$ should be substituted for σ in Eq 8. The problem of defining a proper criterion for brittle fracture under mixed mode loading, and thus specifying an equivalent stress for a multiaxial stress state, is still subject to debate.[13-15] As a first approximation, the equivalent stress can be taken to be the highest principal stress. This is plausible at least in cases where the highest principal stress is considerably larger than the others. However, Griffith's criterion and the weakest link hypothesis, as underlying assumptions, are applicable only to tensile stresses. In the case that absolute values of compressive stress components are several times higher than tensile components, a completely different failure criterion has to be considered.

3.2 Consideration of Subcritical Crack Growth

Of all time-dependent strength degradation processes in ceramics and glasses, subcritical crack growth is most important and has to be considered in the assessment of failure probability.

Phenomenologically, the rate of the SCCG can be approximated (from above) by the power low

$$da/dt = CK_I^n \quad \text{(Eq 9)}$$

where C and n are material parameters, (strongly) dependent on temperature and environmental conditions, and especially on the moisture content of the environment. Susceptibility to SCCG decreases with increasing values of n. For engineering ceramics the exponent n is higher than 30 and can even exceed 200.[16]

By integrating Eq 9 (recalling that $K_I = Y\sigma a^{1/2}$), it is possible to calculate the time a crack of initial length a_i under the action of a constant stress σ would need to reach the critical length a_c, and cause fracture. As a result of SCCG, the number of critical flaws N_c thus increases with time since the lower limit of the inner integral in Eq 6 becomes time-dependent, i.e., $a_i = a_i(t)$. Taking this into account in deriving the formula for the probability of failure one obtains

$$P_f = 1 - \exp\left(-\frac{1}{V_0} \int_V \left(\frac{\langle\overline{\sigma}\rangle}{\sigma_0} \right)^m \left(1 + \frac{\langle\overline{\sigma}\rangle^2}{C_1} t \right)^{m/(n-2)} dV \right) \quad \text{(Eq 10)}$$

with $C_1 = 2/[C(n-2)\,Y^2 K_{Ic}{}^{n-2}]$. Equation 8 for the probability of failure immediately after the application of load appears as a special case of Eq 10 for $t = 0$. The symbol for equivalent stress, $\overline{\sigma}$, is introduced to explicitly allow for the possibility of dealing with multiaxial stress states.

4. Peculiarities of Designing With Brittle Materials

The character and form of Eq 10 for the probability of failure, on which design calculations should be based, allows us to comprehend and summarize the following peculiarities of designing with brittle materials:

- The calculation of strength is based on a *probabilistic evaluation.* Each level of applied stress is associated with a certain probability of failure.
- The integration over the volume of the component brings about the size effect: at a given stress level the probability of failure of a component increases with the size of the component.
- Tensile stresses are decisive for the risk of brittle failure. In the case of a multiaxial stress state, possible compressive stress components enter into the calculation of an appropriately defined equivalent stress. For exceptionally high compressive stresses, however, the calculation of strength has to be based on completely different premises. In any case, in terms of strength, brittle materials are more appropriate for components loaded mainly in compression, whereas tensile stresses should be minimized by appropriate design.
- Since the Weibull modulus m, which typically takes high values, appears as an exponent in Eq 10, the probability of failure is very sensitive to small variations in the highest tensile stresses that arise. Therefore, the distribution of stresses should be kept as uniform as possible, and design-conditioned stress concentrations must be avoided in any case.
- Since stresses at every point in the component enter into the calculation of strength, a precise knowledge of stress distribution over the volume is required. This can generally be achieved only by means of computer aided numerical methods of analysis. Most often the finite element method is used.
- The probability of failure of mechanically loaded ceramic components increases with time. The phenomenon of subcritical crack growth, which is responsible for this, should be considered in the calculation of strength.

5. Brittle Material Data for Design Calculation

Worldwide efforts during the last three decades in the development of structural ceramics resulted in greatly improved performance of these materials. Most of them now reach more than twice the strength measured in the 1970s.[17] The process technologies have become less expensive and more reliable, so that Weibull moduli of up to 30 are even measured on sets of samples taken from components.[18] Higher toughness and the employment of various toughening mechanisms have led to more defect tolerant materials. The understanding of the dependence of microstructure and material properties on the properties of raw material and process technology steadily improves, allowing materials to be tailored for specific applications[19]

Properties of ceramics strongly depend on the manufacturing route. The strength, for instance, as explained above, directly depends on the size of the critical flaw, which can arise in almost any process step: inclusions can be trailed in with raw material, pores or badly sintered regions can result from bad mixing of ingredients, density gradients occurring in the production of green bodies may cause cracks or pores to appear, and the same may result from inappropriate sintering conditions.

Some simple design studies have been performed using typical values for relevant material properties to demonstrate the principal suitability of engineering ceramics for mechanically loaded components.[20] These studies clearly show the crucial influence of material properties on the long-term reliability of the components. However, data that characterize these properties are still not included in manufacturers' data sheets.

Only a few ceramic materials have been thoroughly studied with respect to all kinds of properties, for example the silicon nitride NC132.[21-23] But often these investigations were aimed at other objectives and have to be considered unsystematic with regard to the usability of these data for design purposes. A consequence of this is a serious lack of data appropriate for design with ceramics. Good reviews of existing data are rare.[16,24] Thus in most cases, and especially if subcritical crack growth, fatigue or high temperature are to be considered (as they often are), relevant design data for the time being have to be determined by designers themselves. This has been nicely shown in the case of the silicon nitride turbo charger rotor.[25] To improve this situation the European Structural Integrity Society (ESIS) recently started a concerted action to determine all design relevant data of a typical commercial silicon nitride material, first results being already reported.[26-28]

To give an example, Table 1 presents a collection of properties of three classes of engineering ceramics, which are suitable for structural applications. Alumina ceramics are very hard and wear resistant, they have a medium strength for a low price, and their thermal shock resistance is very low. Silicon carbide ceramics are a good choice if high strength at very high temperatures is required. They are much more thermal shock resistant than alumina ceramics. Silicon nitride ceramics have high strength at room temperature, high toughness, and very high thermal shock resistance. For many structural applications they are the best choice, but they are relatively expensive. Typical values for material properties are given in Table 1, which are relevant for commercial ceramics and should only give a rough estimate. The range of the data is often relatively wide, since cheaper qualities can also be suitable for less demanding applications. Extreme values, as those sometimes reported by various groups of researchers, which cannot be expected from commercial materials, have not been considered (for example, for nano-particle reinforced silicon nitrides, flexural strength values of more than 1300 MPa were reported).[29]

Attention should be given to the reported values for the strength of ceramic materials. Usually, and in Table 1 as well, flexural (or bending) strength is given, which is obtained by applying the Weibull statistics to the maximum bending stresses at fracture of a number of equal specimens. The probability of failure, however, as can be seen from Eq 8, depends on the Weibull modulus m, the size of the specimen, and on the stress distribution over the specimen. Thus the characteristic strength σ_0, to be applied in the design calculations, and which refers to the uniaxially loaded reference volume V_0, has to be calculated first. The conversion formula is obtained by equalizing the probabilities of failure in both situations.[2] If four-point bending with the load applied at the quarter points of

Table 1 Properties of Commercial Engineering Ceramics

Material	Property	Unit	Range	Comments
High alumina	Density, ρ	g/cm^3	3.7-4.0	
	Hardness, Vickers	[HV 5]	1200-1800	
	Modulus of elasticity, E	GPa	300-390	Shows only a slight decrease from room temperature (RT) to up to 1300 °C. For alumina containing a glassy grain boundary phase, it strongly decreases above the glass transition temperature, which generally lies above 700 °C.
	Poisson's ratio, v	1	0.21-0.24	
	Flexural strength, four-point bending, σ_{4b}	MPa	250-400	For very pure alumina it does not change with temperature from RT to up to 1300 °C. For alumina containing a glassy grain boundary phase, it strongly decreases above the glass transition temperature.
	Weibull modulus, m	1	10-30	Does not change with temperature.
	Subcritical crack growth exponent, n	1	30-200	Strongly decreases with temperature. Higher values belong to pure alumina and lower values to alumina containing glassy phases.
	Fracture toughness, K_{Ic}	$MPa \cdot m^{1/2}$	3-5	Values obtained with indentation methods may be higher.
Sintered silicon carbide	Density, ρ	g/cm^3	3.0-3.2	
	Hardness, Vickers	[HV 5]	1800-2500	
	Modulus of elasticity, E	GPa	380-420	Shows only a slight decrease from RT to up to 1400 °C.
	Poisson's ratio, v	1	0.15-0.17	
	Flexural strength, four-point bending, σ_{4b}	MPa	350-600	Almost independent of temperature from RT to up to 1400 °C.
	Weibull modulus, m	1	10-15	Does not change with temperature.
	Subcritical crack growth exponent, n	1	around 200	Decreases with temperature.
	Fracture toughness, K_{Ic}	$MPa \cdot m^{1/2}$	around 3	Values obtained with indentation methods may be higher.
Gas-pressure sintered silicon nitride	Density, ρ	g/cm^3	3.1-3.3	
	Hardness, Vickers	[HV 5]	1200-1600	
	Modulus of elasticity, E	GPa	280-320	Shows only a slight decrease from RT to up to 800 °C.
	Poisson's ratio, v	1	0.24	
	Flexural strength, four-point bending, σ_{4b}	MPa	600-1100	High quality materials should have a flexural strength higher than 900 MPa. Strength decreases with temperature above the glass transition temperature, which generally lies above 800 °C.
	Weibull modulus, m	1	10-30	Now values higher than 15 are normal. It does not change with temperature.
	Subcritical crack growth exponent, n	1	around 50	Strongly decreases with temperature.
	Fracture toughness, K_{Ic}	$MPa \cdot m^{1/2}$	4-8	Values obtained with indentation methods may be higher.

prismatic bars with the rectangular cross section has been applied for the determination of the flexural strength σ_{b4}, the characteristic strength is given by

$$\sigma_0 = \sigma_{b4} \cdot \frac{\left(\frac{V}{V_0}\right)^{1/m}\left[\frac{m+2}{4(m+1)^2}\right]^{1/m}}{\Gamma\left(1+\frac{1}{m}\right)} \quad \text{(Eq 11)}$$

where V is the stressed (gage-length) volume of the specimen and Γ is the gamma-function. For instance, for a standardized four-point bending specimen 40 mm long, 4 mm wide, and 3 mm high, a Weibull modulus of $m = 15$, and a chosen reference volume of $V_0 = 1$ mm^3, the characteristic strength σ_0 is about 19% greater than the flexural strength σ_{b4}. If three-point bending is applied to prismatic bars with a rectangular cross section to determine the flexural strength σ_{b3}, the characteristic strength is given by

$$\sigma_0 = \sigma_{b3} \cdot \frac{\left(\frac{V}{V_0}\right)^{1/m}\left[\frac{1}{2(m+1)^2}\right]^{1/m}}{\Gamma\left(1+\frac{1}{m}\right)} \quad \text{(Eq 12)}$$

By comparing Eq 11 and 12, it can be seen that the same value of the characteristic strength σ_0 is obtained with flexural

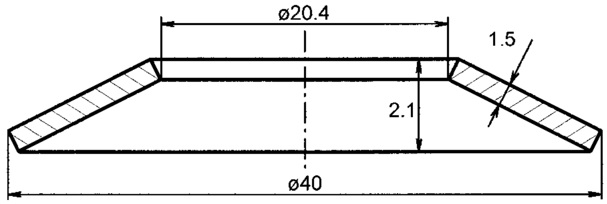

Fig. 1 The ceramic disc spring (height not to scale)

strength σ_{b3} higher than σ_{b4}. Some manufacturers like to report about flexural strength obtained by three-point bending, since they obtain higher numbers.

6. An Example—The Design Calculation of a Ceramic Disc Spring

The above stated peculiarities of designing with a brittle material will be illustrated by the example of the probabilistic calculation of strength of a disc spring made of standard silicon nitride (Si_3N_4). The design of the spring is shown in Fig. 1. Typical values of material parameters are assumed: elastic constants $E = 300$ GPa and $\nu = 0.27$, Weibull parameters $m = 15$ and $\sigma_0 = 1200$ MPa for a unit volume $V_0 = 1$ mm^3, and subcritical crack growth parameters $n = 50$ and $C = 10^5$ MPa^{2}s.

Since the exact distribution of stresses over the volume needs to be known for the calculation of strength of a ceramic spring, the application of the fully geometrically nonlinear theory of thin elastic conical shells may be needed for this purpose.[30] For a shallow spring—i.e., a shallow shell such as the one under consideration, that undergoes small deflexions compared with its dimensions—however, the linear theory is applicable as well, which gives a linear dependence of deflexion and stresses on the applied load. Conveniently, there exists an analytical (although rather complicated) solution to the linearized problem of elasticity of thin conical shells.[31] Such an exceptional case has advantages for further analysis, which can then remain analytical. Figure 2 shows the distribution of stresses over the radial section of the disc spring in Fig. 1, modeled as a conical shell simply supported around the outer edge and loaded with an axial force F uniformly distributed along the inner edge. A force of about 2400 N would be needed to flatten the spring. The variation of the meridian and circumferential stresses, σ_x and σ_φ respectively, over the thickness is linear and that of the shear stress τ_{xz} is parabolic. The circumferential stresses predominate over the largest part of the section. The highest tensile stresses reach only one half of the absolute value of the maximum compressive stress and are more or less uniform, both of which are beneficial for a ceramic spring. Since the circumferential stress is at the same time one of the principal stresses, and the others are comparatively small, it can be used as the equivalent stress for the calculation of failure probability according to Eq 10. This approximation is of negligible consequence for the result, whichever fracture criterion and definition of equivalent stress for a multiaxial stress state is applied.

The results of the calculation of strength of the spring with respect to the magnitude and duration of the applied loading are presented in Fig. 3 and 4. The diagram in Fig. 3 shows the probability of survival, as the complement to unity of the probability of failure $P_s = 1 - P_f$, immediately after the application of load and after intervals of time (from 10^2 to 10^8 s) spent under an invariable load. The decrease of the survival probability with time due to SCCG can be clearly seen from the logarithmic plot in Fig. 4. Under a load of 1800 N, approximately corresponding to the maximum conventionally allowable design deflexion of 3/4 of the clearance height of the spring, 99% of springs will survive for a very short time, 96% will endure 100 s, whereas only 1 in 20 will remain after 10^8 s. The acceptable level of reliability depends on the specific assignment of the spring; for 99% reliability to be maintained after 10^8 s (more than 3 years) spent under the maximum load, the allowable load reduces to 1250 N.

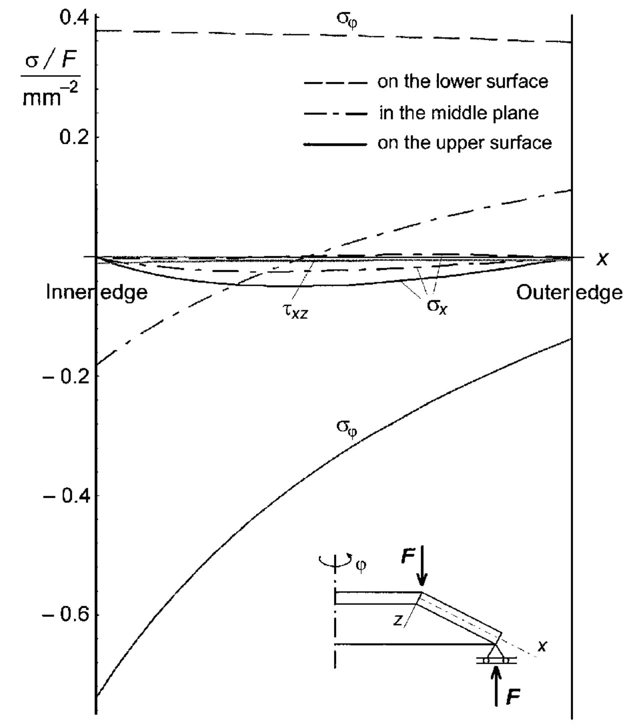

Fig. 2 The distribution of stresses, normalized by applied force, over the radial section of the disc spring

In practice, more detailed stress analysis than that used for the illustrative purposes here might be necessary to take into account the influence of the exact edge geometry and concentrated load application along the edge. To this end the use of numerical methods is indispensable. For service at high temperatures the effects of creep and even cyclic fatigue may need to be considered. The reliability assurance of a ceramic spring would normally also include an adequate proof testing.[12,32]

7. Summary

The stochastic nature of brittle fracture originates from the random occurrence of inevitable microstructural defects in the material. By acting as stress concentrators they allow the ini-

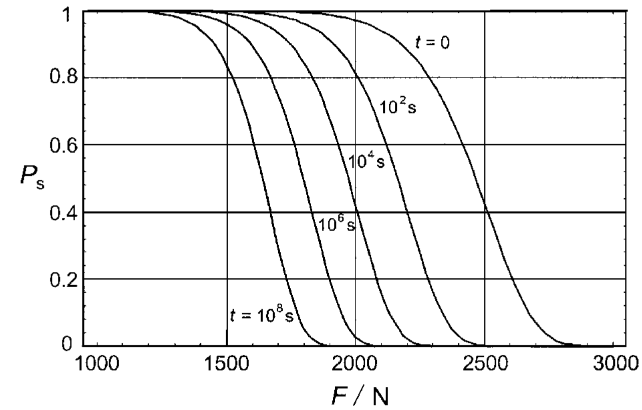

Fig. 3 The probability of survival of the ceramic disc spring vs an applied load after different times

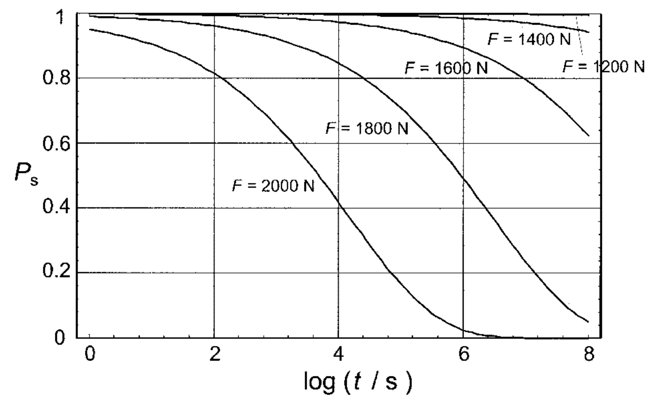

Fig. 4 The probability of survival of the ceramic disc spring vs the time spent under loads of different magnitude

tiation of cracks, one of which may suddenly extend and result in fracture. To account for this, a special approach to design with brittle materials based on the application of principles of probabilistic fracture mechanics and essentially different from that appropriate to conventional, ductile structural materials, is required.

Tensile stresses are decisive for the risk of brittle failure. In terms of strength, brittle materials are more appropriate for components loaded mainly in compression, whereas tensile stresses should be minimized by appropriate design. Since stresses at every point of the component enter into the calculation of strength, a precise knowledge of stress distribution over the volume of the component is required. The probability of failure of a brittle component is very sensitive to small variations in the highest tensile stresses that arise. At a given stress level the probability of failure increases with the size of the component. Furthermore, the probability of failure of mechanically loaded ceramic components increases with time due to the phenomenon of subcritical crack growth.

As has been illustrated by the example of a ceramic disc spring, a proper approach to design with brittle materials can ensure the reliability of ceramic components for load-bearing applications. The troublesome occurrence of delayed fracture due to subcritical crack growth in ceramics, to which the majority of failures of components made of these materials may be attributed, can also be adequately accounted for in the assessment of reliability.

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